Exercise 33

(a) The curve with equation $y^2 = 5x^4 - x^2$ is called a **kampyle of Eudoxus**. Find an equation of the tangent line to this curve at the point $(1, 2)$.

(b) Illustrate part (a) by graphing the curve and the tangent line on a common screen. (If your graphing device will graph implicitly defined curves, then use that capability. If not, you can still graph this curve by graphing its upper and lower halves separately.)

Solution

The aim is to evaluate y' at $x = 1$ and $y = 2$ in order to find the slope there. Differentiate both sides of the given equation with respect to x.

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(5x^4 - x^2)$$

$$(2y) \cdot \frac{d}{dx}(y) = 5\frac{d}{dx}(x^4) - \frac{d}{dx}(x^2)$$

$$(2y) \cdot y' = 5(4x^3) - (2x)$$

Solve for y'.

$$y' = \frac{20x^3 - 2x}{2y} = \frac{10x^3 - x}{y}$$

Evaluate y' at $x = 1$ and $y = 2$.

$$y'(1, 2) = \frac{10(1)^3 - (1)}{(2)} = \frac{9}{2}$$

Therefore, the equation of the tangent line to the curve represented by $y^2 = 5x^4 - x^2$ at $(1, 2)$ is

$$y - 2 = \frac{9}{2}(x - 1).$$

Below is a graph of the curve and the tangent line at $(1, 2)$.